ON Fuzzy soft Baire spaces

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ABSTRACT-- Some new concepts of Baireness in fuzzy soft topological spaces are introduced, and their characterizations and properties are investigated in this work. In continuation of earlier work fuzzy soft nowhere dense set we further investigate several properties and characterizations of fuzzy soft Baire spaces.

Keywords: Fuzzy soft dense, fuzzy soft nowhere dense, fuzzy soft first category, fuzzy soft second category, fuzzy soft G_δ-set, fuzzy soft Baire space.

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1. Introduction

The concept of Baire spaces have been studied extensively in classical topology in [1]. In 2013 the concept of baire space in fuzzy setting was introduced and studied G.Thangaraj and s.Anjalmose[5]. Later other authors like Maji et al.[3-4] have further studied the theory of soft sets and used this theory to solve some decision making problems.

Next, the concept of fuzzy soft set is introduced and studied [2] a more generalized concept, which is a combination of fuzzy set and soft set and studied its properties. Since then much attention has been paid to generalize the basic concepts of fuzzy topology in soft setting and this modern theory of fuzzy soft topology has been developed. In recent years, fuzzy soft topology has been found to be very useful in solving any practical problems. The aim of this paper is to introduce the concepts of fuzzy soft Baireness in fuzzy soft topological spaces. In this paper we discuss several characterizations of fuzzy soft dense, fuzzy soft G_δ- set, fuzzy soft nowhere dense, fuzzy soft first category, fuzzy soft second category, fuzzy soft Baire spaces.

2. preliminaries

Definition 2.1:

The fuzzy soft set $F_\phi \in FS$ (U, E) is said to be null fuzzy soft set and it is denoted by ϕ, if for all $e \in E$, $F(e)$ is the null fuzzy soft set $\bar{0}$ of U, where $\bar{0}(x) = 0$ for all $x \in U$.

Definition 2.2:

Let $FE \in$FS $(U,$ E) and $FE(e) = \bar{1}$ all $e \in E$, where $\bar{1}(x) = 1$ for all $x \in U$. Then F_E is called absolute fuzzy soft set. It is denoted by $\bar{E}$.

Definition 2.3:

A fuzzy soft set FA is said to be a fuzzy soft subset of a fuzzy soft set GB over a common universe U if $A \subseteq B$ and F_A $(e) \subseteq G$B(e) for all $e \in A$, *i.e.*, if $\mu^e{}_{F_A}$ $(x) \leq \mu^e{}_{GB}$ (x) for all $x \in U$ and for all $e \in E$ and denoted by $F_A \tilde{\subseteq} GB$.

Definition 2.4:

Two fuzzy soft sets F_A and G_B over a common universe U are said to be fuzzy soft equal if FA is a fuzzy soft subset of G_B and G_B is a fuzzy soft subset of F_A.

Definition 2.5:

The union of two fuzzy soft sets FA and GB over the common universe U is the fuzzy soft set HC, defined by $H_C(e) = \mu^e_{H_C} = \mu^e_{F_A} \cup \mu^e_{G_B}$ for all $e \in E$, where $C = A \cup B$. Here we write $H_C = F_A \tilde{\vee} G_B$.

Definition 2.6:

Let F_A and G_B be two fuzzy soft set, then the intersection of F_A and G_B is a fuzzy soft set H_C, defined by $H(e) = \mu^e_{F_C} = \mu^e_{F_A} \cap \mu^e_{G_B}$ for all $e \in E$, where $C = A \cap B$. Here we write $H_C = F_A \tilde{\wedge} G_B$.

Definition 2.7:

Let $F_A \in$FS $(U,$ E) be a fuzzy soft set. Then the complement of F_A, denoted by F_A^C, defined by

$$F_A^C(e) = \begin{cases} \bar{1} - \mu^e_F A, & if \;\; e \in A \\ \bar{1}, & if e \notin A \end{cases}$$

Definition 2.8:

Let ψ be the collection of fuzzy soft sets over U. Then ψ is called a fuzzy soft topology on U if ψ satisfies the following axioms:

(i) $\phi, \bar{E}$ belong to ψ.

(ii) The union of any number of fuzzy soft sets in ψ belongs to ψ.

(iii) The intersection of any two fuzzy soft sets ψ belongs to ψ. The triplet (U, E, ψ) is called a fuzzy soft topological space over U. The members of ψ are called fuzzy soft open sets in U and complements of them are called fuzzy soft closed sets in U.

Definition 2.9:

The union of all fuzzy soft open subsets of F_A over (U, E,ψ) is called the interior of F_A and is denoted by $int^{fs}(F_A)$.

Definition 2.10:

The union of all fuzzy soft open subsets of F_A over (U, E) is called the interior of F_A and is denoted by $int^{fs}(F_A)$.

Definition 2.11:

Let $F_A \in FS(U,E)$ be a fuzzy soft set. Then the intersection of all closed sets, each containing F_A, is called the closure of F_A and is denoted by $cl^{fs}(F_A)$.

3. FUZZY SOFT BAIRE SPACES

Definition 3.1:

Let (U,E,ψ) be a fuzzy soft topological space. Then (U,E,ψ) is called a fuzzy soft baire space if $int^{fs}(\vee_{i=1}^{\infty}(F_{A_i}))=0$ where F_{A_i} ′s are fuzzy soft nowhere dense sets in (U,E,ψ).

Proposition3.2:

If F_A is a fuzzy soft dense and fuzzy G_δ -set in a fuzzy soft topological space (U, E, ψ), then $1-F_A$ is a fuzzy soft first category set in (U, E, ψ).

Proof:

Since F_A is a fuzzy soft G_δ set in (U, E, ψ), $F_A=\wedge_{i=1}^{\infty}(F_{A_i})$ where $F_{A_i}\in T$ and since F_A is a fuzzy soft dense in (U, E, ψ), $cl^{fs}(F_A)=1$, then $cl^{fs}(\wedge_{i=1}^{\infty}(F_{A_i}))=1$.But $cl^{fs}(\wedge_{i=1}^{\infty}(F_{A_i}))\leq\wedge_{i=1}^{\infty} cl^{fs}(F_{A_i})$.Hence $1\leq\wedge_{i=1}^{\infty} cl^{fs}(F_{A_i})$.(ie) $\wedge_{i=1}^{\infty} cl^{fs}(F_{A_i})=1$. Then we have $cl^{fs}(F_{A_i})=1$ for each $F_{Ai}\in T$ and hence $cl^{fs}(int\, F_{Ai})=1$.which Implies that $1- cl^{fs}\, int^{fs}(F_{A_i})=0$ and hence $int^{fs}\, cl^{fs}(1-F_{A_i})=0$. Therefore $1-F_{A_i}$ is a fuzzy soft nowhere dense set in (U,E,ψ). Now $1-F_{A}=1-\wedge_{i=1}^{\infty}(F_{A_i}) = \vee_{i=1}^{\infty}(1-F_{A_i})$ therefore $1-F_{A}=\vee_{i=1}^{\infty}(1-F_{A_i})$ where $(1-F_{A_i})$′s are fuzzy soft nowhere dense sets in (U, E, ψ).Hence $1-F_{A}$ is a fuzzy soft first category set in (U, E, ψ).

Definition 3.3:

Let F_A be a fuzzy soft first category set in a fuzzy soft topological space (U,E,ψ).Then $1-F_A$ is called a fuzzy soft residual set in (U,E,ψ).

Proposition 3.4:

If F_A is a fuzzy soft dense and fuzzy soft G_δ set in a fuzzy soft topological space (U, E, ψ), then F_A is a fuzzy soft residual set in (U,E,ψ).

Proof:

Since F_A is a fuzzy soft dense and fuzzy soft G_δ set in (U, E, ψ), by proposition 3.2, we have $1-F_A$ is a fuzzy soft first category set in (U,E,ψ) and hence F_A is a fuzzy soft residual set in (U, E, ψ).

Proposition 3.5:

If the fuzzy soft topological space (U, E, ψ) has a fuzzy soft dense and fuzzy soft G_δ set, then (U, E, ψ) is a fuzzy soft Baire space.

Proof:

Let F_A be a fuzzy soft dense and G_δ set in (U, E,ψ). Then by proposition 3.2, $1-F_A$ is a fuzzy soft first category set in (U,E,ψ) and $(1-F_A) =\vee_{i=1}^{\infty}(1-F_{A_i})$ where $(1-F_{A_i})$′s are fuzzy soft nowhere dense sets in (U, E, ψ). But $int^{fs}(1-F_A)=1-cl^{fs}(F_A)=1-1=0$ (since F_A is fuzzy soft dense, $cl^{fs}(F_A)=1$). Then $int^{fs}(\vee_{i=1}^{\infty}(1-F_{A_i}))= int^{fs}(1-F_A) = 0$ and hence (U,E,ψ) is a fuzzy soft baire space.

Proposition 3.6:

If the fuzzy soft topological space (U,E,ψ) is a fuzzy soft first category space, then (U,E,ψ) is not a fuzzy soft baire space.

Proof:

Since (U, E, ψ) is a fuzzy soft first category space,then $\vee_{i=1}^{\infty} F_{A_i} =1$, where F_{A_i} ′s are fuzzy soft nowhere dense sets in (U, E, ψ). Therefore $int^{fs}(\vee_{i=1}^{\infty}(F_{A_i})) = int^{fs}(1) =1\neq0$. Hence $int(\vee_{i=1}^{\infty}(F_{A_i}))\neq0$, where F_{A_i} ′s are fuzzy soft nowhere dense sets in (U, E, ψ) and therefore (U, E, ψ) is not a fuzzy soft Baire space.

Definition 3.7:

Let (U, E, ψ) be a fuzzy soft topological space. Then (U, E, ψ) is called a fuzzy soft baire space if $int^{fs}(\vee_{i=1}^{\infty}(F_{A_i}))=0$, where F_{A_i}′s are fuzzy soft nowhere dense sets in (U, E, ψ).

EXAMPLE 3.8:

Let U={C_1,C_2,C_3} be the set of three flats and E={e_1,e_2,e_3}, where e_1,e_2,e_3 stand for costly, modern, security services. Then we consider

$$F_E=\begin{bmatrix} .6 & .5 & .7 \\ .6 & .4 & .6 \\ .5 & .3 & .4 \end{bmatrix},$$

$$G_E=\begin{bmatrix} .6 & .8 & .7 \\ .4 & .7 & .7 \\ .3 & .6 & .5 \end{bmatrix} H_E=\begin{bmatrix} .6 & .5 & .8 \\ .5 & .3 & .7 \\ .4 & .2 & .6 \end{bmatrix}$$

$T^1_E=F_E\wedge G_E$, $T^2_E=F_E \wedge H_E$, $T^3_E=G_E\wedge H_E$,$T^4_E= F_E\vee G_E$, $T^5_E = F_E\vee H_E$,$T^6_E = G_E\vee H_E$, $T^7_E = F_E\wedge (G_E\vee H_E)$, $T^8_E = F_E\vee(G_E\wedge H_E)$, $T^9_E= G_E\wedge(F_E\vee H_E)$

$T^{10}_E = G_E\vee(F_E\wedge H_E)$,$T^{11}_E= H_E\wedge (F_E\vee H_E)$, $T^{12}_E= H_E \vee(F_E\wedge G_E)$, $T^{13}_E = F_E\vee G_E\vee H_E$ and $T^{14}_E =F_E\wedge G_E\wedge H_E$.

Now consider the fuzzy soft sets α = $(1-F_E)\vee(1- T^8_E) \vee (1- T^{13}_E) =\begin{bmatrix} .6 & .5 & .7 \\ .6 & .6 & .7 \\ .5 & .7 & .6 \end{bmatrix}$,

$\beta = (1-G_E) \vee (1-T^5_E) \vee (1-T^{10}_E) = \begin{bmatrix} .4 & .5 & .3 \\ .6 & .6 & .3 \\ .7 & .7 & .5 \end{bmatrix}$
and $\upsilon = (1-H_E) \vee (1-T^1_E) \vee (1-T^4_E) \vee (1-T^6_E)$
$= \begin{bmatrix} .4 & .5 & .3 \\ .6 & .7 & .4 \\ .7 & .8 & .6 \end{bmatrix}$ in (U, E, ψ). Then α, β, and υ are fuzzy soft sets in (U, E, ψ) and $int^{fs}(\alpha)=0$, $int^{fs}(\beta) = 0$ and, $int^{fs}(\upsilon)=0$. Then α, β, υ are fuzzy soft nowhere dense sets in (U, E,ψ).

Moreover, $(1-T^2_E) \vee (1-T^3_E) \vee (1-T^7_E) \vee (1-T^9_E) \vee (1-T^{11}_E) \vee (1-T^{12}_E) \vee (1-T^{14}_E) = \upsilon$ and also $int^{fs}(\alpha \vee \beta \vee \upsilon) = int^{fs}(\upsilon)=0$ and therefore (U,E,ψ) is a fuzzy soft baire space.

Proposition 3.9:

Let (U, E,ψ) be a fuzzy soft topological space. Then the following are equivalent:

(1)(U, E, ψ) is a fuzzy soft Baire space.

(2)$int^{fs}(F_A)=0$ For every fuzzy soft first category Set in F_A in (U, E,ψ).

(3) $cl^{fs}(G_B)= 1$ for every fuzzy soft residual set G_B In (U,E, ψ).

Proof:

(**1**)⟹(**2**). Let $F_A$be a fuzzy soft first category set in (U, E, ψ).Then $F_A=\vee_{i=1}^{\infty}(F_{A_i})$,where $F_{Ai'}$ s are fuzzy soft nowhere dense sets in (U,E,ψ).Then, we have $int^{fs}(F_A) = int^{fs}(\vee_{i=1}^{\infty}(F_{Ai}))$. Since (U, E, ψ) is a fuzzy soft baire space, $(int^{fs}\vee_{i=1}^{\infty}(F_{Ai}))=0$. Hence $int^{fs}(F_A)=0$ for any fuzzy soft first category set F_A in (U, E, ψ).

(**2**)⟹(**3**). Let G_B be a fuzzy soft residual set in (U, E,ψ). Then $1-G_B$ is a fuzzy soft first category set in (U, E,ψ). By hypothesis, $int^{fs}(1-G_B)=0$. Then $1- cl^{fs}(G_B)=0$. Hence $cl^{fs}(G_B)= 1$ for any fuzzy soft residual set G_B in (U,E, ψ).

(**3**)⟹(**1**). Let F_A be a fuzzy soft first category set in (U, E, ψ).$F_A=\vee_{i=1}^{\infty}(F_{A_i})$,where $F_{A_i'}$ s are fuzzy soft nowhere dense sets in (U,E,ψ). Now F_A is a fuzzy soft first category set in (U, E,ψ) implies that $(1-F_A)$ is a fuzzy soft residual set in (U, E, ψ). By hypothesis, we have $cl^{fs}(1-F_A)=1$. Then $1- int^{fs}(F_A)=1$.Hence $int^{fs}(F_A)=0$. That is, $int^{fs}(\vee_{i=1}^{\infty}(F_{A_i})=0$ where $F_{Ai'}$ s are fuzzy soft nowhere dense sets in (U, E, ψ). Hence (U, E, ψ) is a fuzzy soft baire space.

Definition 3.10:

A fuzzy soft topological space (U, E, ψ) is called fuzzy soft first category space if the fuzzy soft set 1_x is a fuzzy soft first category set in (U, E, ψ). That is ,$1_x = \vee_{i=1}^{\infty}(F_{A_i})$ where $F_{A_i'}$s are fuzzy soft nowhere dense sets in (U, E, ψ). Otherwise (U, E, ψ) will be called a fuzzy soft second category space.

Proposition3.11:

If the fuzzy topological space (U, E, ψ) is a fuzzy soft first category space, then (U, E, ψ) is not a fuzzy soft Baire space .

Proof :

Let the fuzzy soft topological space (U, E, ψ) is a fuzzy soft first category space. Then $\vee_{i=1}^{\infty} F_{A_i}=1$,Where F_{Ai}'sare fuzzy soft nowhere dense sets in (U,E,ψ). Now $int^{fs}(\vee_{i=1}^{\infty}(F_{A_i}) = int^{fs}(1) \neq 0$. Hence by definition, (U, E, ψ) is not a fuzzy soft Baire space.

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